Instructor: Larry Zimmerman

Our Level A course is appropriate for students who have not taken any other NYMC High School Level classes. In addition, students enrolling in Level A should have a good command of Algebra 1 and be familiar with some introductory Geometry. Here are some sample problems from past Level A classes.

1. Find the Highest Common Factor of (a) 124 and 132 (b) 96 and 44 (c) 10125 and 10126.
2. How many distinct positive integer factors has each of the following numbers?
   (a) 4^{15} (b) $(2^{10})(3^5)$ (c) 60^6 (d) $(4^5)(6^4)$
3. How many integers between 10 and 200 have an odd number of divisors?
4. For the number $(2^{10})(3^5)$, compute the number of (a) even factors (b) the number of perfect square factors (c) the number of perfect cube factors.
5. Find ALL possible values of the missing digit A, such that the nine digit number 27545365A is divisible by (a)3 (b)9 (c)6 (d)4 (e)8 (f)11 (g)7.
6. How many multiples of 7 are there between 1 and 1000? How many multiples of 11 are there between 1 and 1000? How many multiples of 7 or 11 are there between 1 and 1000?
7. Find all ordered pairs of positive integers (x, y) such that $x^2-y^2 = 64$. Interpret geometrically.
8. What is the smallest positive integer having precisely
   (a) 13 positive integer divisors? (b) 12 positive integer divisors?
9. Let $N = 10T + U$ be any two digit (base 10) number whose tens digit is T and whose units digit is U. Show that
   (a) If 9 divides $(10T + U)$, then 9 divides $(T + U)$
   (b) If 7 divides $(10T + U)$, then 7 divides $(3T + U)$
   (c) If 7 divides $(10T + U)$, then 7 divides $(5U + T)$.
10. Can $4.83 in total postage be obtained by using only (a) 7 cent stamps and 13 cent stamps? (b) 4 cent stamps and 44 cent stamps? (c) 12 cent stamps and 15 cent stamps? In each case determine a possible solution if it exists.
11. Show that 5 divides $(n^5 - n)$ for every positive integer n.
12. Show that $x^2 - y^2 = 2$ has no solutions in integers.
13. Show that there is some multiple of 97 that consists entirely of the digit 1. That is, there is a multiple of 97 that is a repunit.
14. Show that no cube (of a positive integer) is the sum of eight consecutive positive integers.
15. Some positive integers can be expressed as the sum of consecutive positive integers. For example, $30 = 9 + 10 + 11$ and $31 = 15 + 16$. However, 32 has no such representation. Find

a three digit number that, like 32, cannot be expressed as the sum of consecutive positive integers.

16. How many diagonals does a decagon have?

17. How many arrangements are there of the letters of the word ORANGE are there such that (a) the letters O and R must be together? (b) The letters of O and R may not be next to one another?

18. How many unlike terms are there in the expansion of $(x + y + z)^{10}$?

19. Compute the sum of the numerical coefficients in the expansion of
    (a) $(x + y)^6$ (b) $(x - y)^6$ (c) $(2x - y)^6$.

20. A Pythagorean Right Triangle (PRT) is a right triangle whose sides all have positive integer lengths. Find all PRTs with a leg of (a) 7 (b) 10 (c) 20 (d) 8.

21. Express $234_{(10)}$ in base (a)9 (b)7 (c)5 (d)3 (e)2 (f)11 (g)16.

22. Express each of the following as an equivalent decimal numeral. (That is, express in base 10) (a) $234_{(7)}$ (b) $1234_{(5)}$ (c) $101011_{(2)}$ (d) $23A8_{(11)}$ (e) $123_{(6)}$ (f) $1331_{(5)}$ (g) $.12_{(3)}$ (h) $.101_{(2)}$

23. Find all positive integers, $m > 1$, such that (a) $23 \equiv 5 \pmod{m}$ (b) $100 \equiv 1 \pmod{m}$.

24. Show that if n is an odd integer then $n^2 \equiv 1 \pmod{8}$.

25. Find the remainder when 21000 is divided by 13.

26. Find all (x, y), where x and y are positive integers and $3x + 11y = 801$.

27. Show that the difference of two consecutive cubes is never divisible by 3.